Whiz Kids²

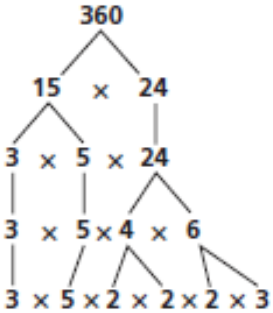
6TH Grade Concepts

School districts have adopted the Connected Math curriculum and the concepts below are directly in line with that curriculum.

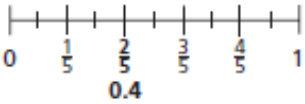
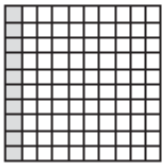
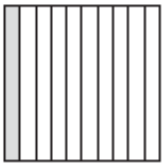
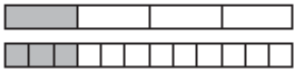
Another helpful resource is www.KhanAcademy.com where you can watch videos explaining each of the following concepts.

Contents

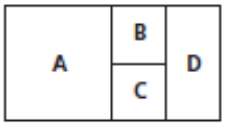
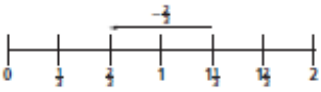
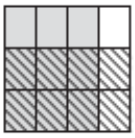
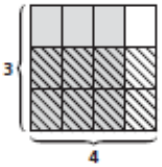
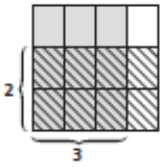
PRIMES: FACTORS AND MULTIPLES

Important Concepts	Examples
Factors One of two or more numbers that are multiplied to get a product.	All the factors of 12 are 1, 2, 3, 4, 6, 12 because $1 \times 12 = 12$, $2 \times 6 = 12$, $3 \times 4 = 12$.
Multiples The product of a given whole number and another whole number.	Some multiples of 12 include 12, 24, 36, 48, 60 because $12 \times 1 = 12$, $12 \times 2 = 24$, $12 \times 3 = 36$, $12 \times 4 = 48$, and $12 \times 5 = 60$. Note that any number has an infinite number of multiples. If a number is a multiple of 12, then 12 is a factor of that number. For example, 36 is a multiple of 12 and 12 is a factor of 36.
Prime A number with exactly two different factors, 1 and the number itself.	Examples of primes are 11, 17, 53, and 101. The number 1 is not a prime number, since it has only one factor. All the factors of 11 are 1 and 11. All the factors of 17 are 1 and 17.
Composite A whole number with factors other than itself and 1 or a whole number that is not prime.	Some composite numbers are 6, 12, 20, and 1,001. Each of these numbers has more than two factors. All the factors of 6 are 1, 2, 3, 6. All the factors of 1,001 are 1, 7, 11, 13, 77, 91, 143, 1001.
Common Multiples A multiple that two or more numbers share.	Some multiples of 5 are 5, 10, 15, 20, 25, 30, <u>35</u>, 40, 45, 50, 55, 60, 65, and <u>70</u>. Some multiples of 7 are 7, 14, 21, 28, <u>35</u>, 42, 49, 56, 63, <u>70</u>, and 77. From these lists we can see that two common multiples of 5 and 7 are 35 and 70. There are more common multiples that can be found.
Common Factors A factor that two or more numbers share.	7 is a common factor of 14 and 35 because 7 is a factor of 14 ($14 = 7 \times 2$) and 7 is a factor of 35 ($35 = 7 \times 5$).
Prime Factorization A product of prime numbers, resulting in the desired number. The prime factorization of a number is unique except for the order of the factors. This is the **Fundamental Theorem of Arithmetic**.	 The prime factorization of 360 is $2 \times 2 \times 2 \times 3 \times 3 \times 5$. Although you can switch the order of the factors, every prime product string for 360 will have three 2s, two 3s, and one 5.

FRACTIONS, DECIMALS, PERCENTS

Important Concepts	Examples
Fractions as Parts of a Whole In the part-whole interpretation of fractions, students must: • determine what the whole is; • subdivide the whole into equal-size parts—not necessarily equal shape, but equal size; • recognize how many parts are needed to represent the situation; and • form the fraction by placing the parts needed over the number of parts into which the whole has been divided.	If there are 24 students in the class and 16 are girls, the part of the whole that is girls can be represented as $\frac{16}{24}$.  The shaded portion above can also be represented as $\frac{2}{3}$.  The **denominator** of 3 tells into how many equal-size parts the whole has been divided, and the **numerator** of 2 tells how many of the equal-size parts have been shaded.
Fractions as Measures or Quantities In this interpretation, a fraction is thought of as a number.	A fraction can be a measurement that is "in between" two whole measures. Students meet this every day in such references as $2\frac{1}{2}$ brownies, 11.5 million people, or $7\frac{1}{2}$ inches.
Fractions as Indicated Divisions To move with flexibility between fraction and decimal representations of rational numbers, students need to understand that fractions can be thought of as indicated divisions.	Sharing 36 apples among 6 people calls for division ($36 \div 6 = 6$ apples each), so sharing 3 apples among 8 people calls for dividing 3 by 8 to find out how many each person receives ($\frac{3}{8}$ of an apple).
Fractions as Decimals Students need to understand decimals in two ways: as special fractions with denominators of 10 and powers of 10, and as a natural extension of the place-value system for representing quantities less than 1.	For the fraction $\frac{2}{5}$, for example, we can find the decimal representation by rewriting as the equivalent fraction $\frac{4}{10}$ or by dividing 2 by 5. This uses the division interpretation of fractions to find the decimal representation of the same quantity. $\frac{2}{5} = 2 \div 5 = 0.4$ 
Fractions as Percents This builds the connection between and among fractions, decimals, and percents. Percents are introduced as special names for hundredths, $\frac{1}{100}$.	Ten percent, 10%, is simply another way to represent 0.10 or 0.1, which is another way to represent $\frac{10}{100}$ or $\frac{1}{10}$.  $\frac{10}{100}$ or 0.10  $\frac{1}{10}$ or 0.1
Equivalent Fractions Partitioning and then partitioning again is an important skill that contributes to understanding equivalence. Equivalent fractions have the same value.	If a bar is marked into fourths (the first partition) and then each fourth is marked into thirds (the second partition), each original fourth has three parts (or three-twelfths) in it. This one-fourth is equivalent to three-twelfths. $\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$ 

FRACTION OPERATIONS

Important Concepts	Examples
Addition and Subtraction of Fractions Students model problems to develop meaning and skill in addition and subtraction. Students learn to find common denominators so that the numerators can be added or subtracted.	To find the sum of A + B on the rectangle, or $\frac{1}{2} + \frac{1}{8}$, students need to use equivalent fractions to rename $\frac{1}{2}$ as $\frac{4}{8}$. The area model helps students visualize A, $\frac{1}{2}$, as $\frac{4}{8}$ and they write the number sentence, $\frac{4}{8} + \frac{1}{8} = \frac{5}{8}$.  The *number-line model* helps connect fractions to quantities. This illustrates $1\frac{1}{3} - \frac{2}{3} = \frac{2}{3}$. 
Developing a Multiplication Algorithm Students use models to see that they can just multiply the numerators and multiply the denominators of proper fractions.	An area model can show $\frac{2}{3} \times \frac{3}{4}$. Shade a square to show $\frac{3}{4}$. To represent taking $\frac{2}{3}$ of $\frac{3}{4}$, cut the square into thirds the opposite way and use hash marks on two of the three sections. The overlap sections represent the product, $\frac{6}{12}$.  The **denominators** partition and repartition the whole. Breaking the fourths into three parts each makes 12 pieces. In the algorithm, you multiply the denominators (3 × 4) to resize the whole to have the correct number of parts.  denominator → $\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12}$  numerator → $\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12}$ The **numerator** is keeping track of how many of the parts are being referenced. You need to take 2 out of 3 sections from each part. This can be represented by the product of the numerators 2 × 3.
Developing a Division Algorithm Students may have various ways to think about division of fractions. Our goal in the development of algorithms is to help students develop an efficient algorithm.	**Common Denominator Approach** Students rewrite $\frac{7}{9} \div \frac{1}{3}$ as $\frac{7}{9} \div \frac{3}{9}$. The common denominator allows the reasoning that if you have 7 one-ninth-sized pieces and want to find out how many groups of 3 one-ninth-sized pieces you can make, then $\frac{7}{9} \div \frac{3}{9}$ has the same answer as $7 \div 3 = 2\frac{1}{3}$. **Multiplying by the Denominator and Dividing by the Numerator** With $9 \div \frac{1}{3}$, you can reason: I have to find the total number of $\frac{1}{3}$s in 9. There are three $\frac{1}{3}$s in 1, so there are 9 × 3, or 27, $\frac{1}{3}$s in 9. $9 \div \frac{1}{3} = 9 \times 3 = 27$. With $\frac{2}{3} \div \frac{1}{5}$, we can reason that $1 \div \frac{1}{5}$ is 5, as $\frac{2}{3} \div \frac{1}{5}$ should be $\frac{2}{3}$ of this, or $\frac{2}{3}$ of 5, or $\frac{10}{3}$. We could also rename $\frac{2}{3} \div \frac{1}{5}$ as $\frac{10}{15} \div \frac{3}{15}$ and see this as 10 fifteenths divided by 3 fifteenths, which is the same as $10 \div 3$, or $\frac{10}{3}$. Notice that this requires us to multiply the number of $\frac{2}{3}$s by 5. With $\frac{2}{3} \div \frac{4}{5}$, we can reason that this should be $\frac{1}{4}$ of $\left(\frac{2}{3} \div \frac{1}{5}\right)$, or $\frac{1}{4}\left(\frac{10}{3}\right) = \frac{10}{12}$. Notice that this reasoning requires us to multiply the denominator of $\frac{2}{3}$ by 4. In short, to compare $\frac{2}{3} \div \frac{4}{5}$ we compute $\frac{2}{3} \times \frac{5}{1} \times \frac{1}{4} = \frac{10}{12}$. That is, we multiply the numerator of $\frac{2}{3}$ by 5 and the denominator by 4. **Multiplying by the Reciprocal** We see that $\frac{2}{3} \div \frac{4}{5}$ (see above) gives the same result as $\frac{2}{3} \times \frac{5}{4}$.

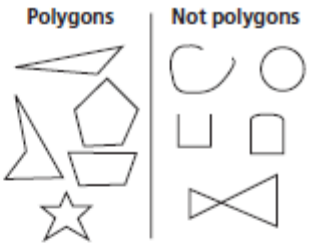
DECIMAL OPERATIONS

Important Concepts	Examples
Addition and Subtraction of Decimals **DECIMALS AS FRACTIONS** Write decimals as fractions, find common denominators, add or subtract the fractions, and express the answers as decimals. This confirms that when adding or subtracting, one must compute with digits of the same place value. **PLACE-VALUE INTERPRETATION** Students consider the place value of digits and what that means when adding or subtracting numbers.	Zeke buys cider for \$1.97 and donuts for \$0.89. The clerk said the bill was \$10.87. What did the clerk do wrong? The cider is \$1.97 $= \frac{197}{100}$ and the donuts are \$0.89 $= \frac{89}{100}$. So the cost is $\frac{197}{100} + \frac{89}{100} = \frac{286}{100} = 2.86$. In $1.97 + 0.89$, we add hundredths to hundredths ($1.9\underline{7} + 0.8\underline{9}$), tenths to tenths ($1.\underline{9}7 + 0.\underline{8}9$), and ones to ones ($\underline{1}.97 + \underline{0}.89$). The clerk incorrectly added dollars and pennies (ones and tenths, tenths and hundredths).
Multiplication of Decimals **DECIMALS AS FRACTIONS** Write decimals as fractions, multiply, write the answer as a decimal, and relate the number of decimal places in the factors to the answer. **PLACE-VALUE INTERPRETATION** Students see why counting decimal points make sense and use the short-cut algorithm: multiply the decimals as whole numbers and adjust the place of the decimal in the product.	We can look at a problem using equivalent fractions. $0.3 \times 2.3 = \frac{3}{10} \times 2\frac{3}{10} = \frac{3}{10} \times \frac{23}{10}$ The product as a fraction is $\frac{69}{100}$, as a decimal 0.69. The 100 in the denominator shows that there should be two decimal places (hundredths) in the answer. The denominator of the fraction tells us the place value needed in the decimal. Using the fact that $25 \times 31 = 775$ students reason about a related product: 2.5×0.31 (2.5 is a tenth of 25, 0.31 is a hundredth of 31, so the product is a thousandth of 775) $= 0.775$.
Division of Decimals **DECIMALS AS FRACTIONS** Write decimals as fractions with common denominators and divide the numerators. **PLACE-VALUE INTERPRETATION** Write an equivalent problem by multiplying both the dividend and the divisor by the same power of ten until both are whole numbers.	$3.25 \div 0.5 = \frac{325}{100} \div \frac{5}{10} = \frac{325}{100} \div \frac{50}{100} = 325 \div 50 = 6.5$ $37.5 \div 0.015 = \frac{375}{10} \div \frac{15}{1{,}000} = \frac{37{,}500}{1{,}000} \div \frac{15}{1{,}000} = 37{,}500 \div 15 = 2{,}500$ This makes a whole number problem with the same quotient as the original decimal problem. The fraction approach explains why moving decimal places works. $0.015\overline{)37.5} = 0.015 \times 1{,}000\overline{)37.5 \times 1{,}000} = 15\overline{)37500}$
Decimal Forms of Rational Numbers **FINITE (OR TERMINATING) DECIMALS** are decimals that "end." The simplified fraction has prime factors of only 2s or 5s in the denominator. **FNFINITE REPEATING DECIMALS** are decimals that "go on forever" but show a repeating pattern. These fractions have prime factors other than 2 or 5 in the simplest denominator form.	$\frac{1}{2} = 0.5$, $\frac{1}{8} = 0.125$, $\frac{12}{75} = 0.16$. $\frac{4}{25} = \frac{16}{100} = 0.16$ In simplified fraction form $\frac{12}{75} = \frac{4}{25}$ has only factors of five $\left(\frac{4}{5 \times 5}\right)$ in the denominator. $\frac{1}{3} = 0.333\ldots$, $\frac{2}{3} = 0.666\ldots$, $\frac{8}{15} = 0.533\ldots$, $\frac{3}{7} = 0.42857142\ldots$ In simplified fraction form $\frac{26}{150} = \frac{13}{75} = \frac{13}{3 \times 5 \times 5} = 0.1733333\ldots$.
Using Percents **PERCENT OF A PRICE** "A CD costs \$7.50. The sales tax is 6%. How much is the tax?" **ON WHAT AMOUNT THE PERCENT WAS FIGURED** "Customers left Jill \$2.50 as a tip. The tip was 20% of the total. How much was the bill?" **WHAT PERCENT ONE NUMBER IS OF ANOTHER NUMBER** "Sam got a \$12 discount off a \$48 purchase. What percent discount did he get?"	6% of \$7.50 = *cost of tax* 1% of \$7.50 $= \frac{1}{100}$ of \$7.50 = \$7.50 ÷ 100 = 0.075 6 of the 1%'s will give me 6%. So, 6% of \$7.50 = \$0.45. 20% of *some number* equals \$2.50 Find how many 20%s it takes to make 100%. In this case we need five. So, 5 × \$2.50 gives us \$12.50. Find *what* % 12 is of 48. Students can solve this by computing how many 12s in 48. It takes four, so the percent is $\frac{1}{4}$ of 100% or 25%.

TWO-DIMENSIONAL GEOMETRY

Important Concepts and Examples

Polygon
A shape formed by line segments so that each of the segments meets exactly two other segments, and all of the points where the segments meet are end points of the segments.

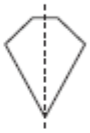


Polygon Names
Triangle 3 sides and 3 angles
Quadrilateral 4 sides and 4 angles
Pentagon 5 sides and 5 angles
Hexagon 6 sides and 6 angles
Heptagon 7 sides and 7 angles
Octagon 8 sides and 8 angles
Nonagon 9 sides and 9 angles
Decagon 10 sides and 10 angles
Dodecagon 12 sides and 12 angles

Regular Polygons
Polygons whose side lengths are equal and interior angle measures are equal.

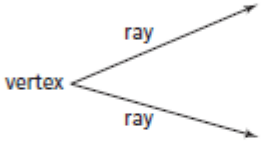
Irregular Polygon
A polygon that has either at least two sides with different lengths or two angles with different measures

Line (or Mirror) Symmetry
If the polygon is folded over the line of symmetry, the two halves of the shape will match exactly.



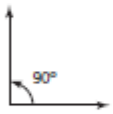
Rotational (or Turn) Symmetry
A polygon with turn symmetry can be turned around its center point less than a full turn and still look exactly as it did before it was rotated.

Angles
Angles are figures formed by two rays or line segments that have a common vertex. The **vertex** of an angle is the point where the two rays meet or intersect. Angles are measured in degrees.

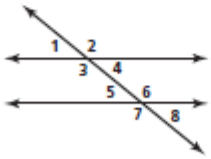


Angle Measures
To develop estimation skills, students relate angles to right angles. Combinations and partitions of 90° are used as benchmarks to estimate angle size.

A **goniometer** (goh nee AHM uh tur), or **angle ruler**, is an instrument for making more precise measurements of angles. This tool is used in the medical field for measuring angle of motion or flexibility in body joints, such as knees.



Angles and Parallel Lines
Parallel lines cut by a **transversal** make pairs of equal corresponding angles and pairs of equal alternate interior angles. Angles 1 and 5, angles 2 and 6, angles 3 and 7, and angles 4 and 8 are pairs of **corresponding angles**. Angles 4 and 5 and angles 3 and 6 are pairs of **alternate interior angles**.

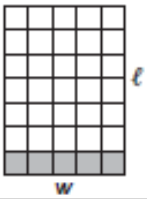
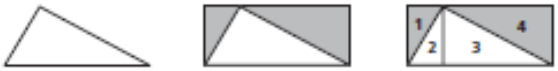
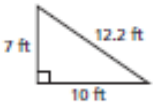
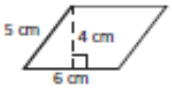
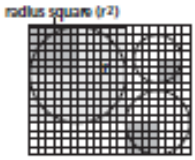


Polygons That Tile a Plane
For regular polygons to tile a plane (or cover a flat surface without gaps or overlaps), the angle measure of an interior angle must be a factor of 360°. The only regular polygons that can tile a plane are an equilateral triangle (60° angles), a square (90° angles), and a regular hexagon (120° angles).

Triangle Inequality Theorem
The sum of two side lengths of a triangle must be greater than the 3rd side length.

If the side lengths are a, b, and c, then: $a + b > c$, $b + c > a$, $c + a > b$

MEASURING GEOMETRIC SHAPES

Important Concepts	Examples
The Measurement Process • Identify an object and the attribute to be measured. • Select an appropriate unit. • Repeatedly "match" the unit to the attribute of the object (or phenomenon, such as time). • Determine the number of units.	**Measuring Perimeter** Measuring perimeter requires counting how many linear units are needed to surround an object. **Measuring Area** Measuring area requires counting how many square units are needed to cover an object.
Area of Rectangles Students begin finding the area by counting the number of squares enclosed. To count more efficiently, they find the number of squares in one row and multiply by the number of rows. In other words, find the area by multiplying the length (how many in a row) by the width (the number of rows).	There are 5 squares in the first row and 7 rows in all. The area of the rectangle is $5 \times 7 = 35$ square units or, in general, $\ell \times w$. 
Perimeter of Rectangles Students count the number of linear units surrounding the rectangle. To count more efficiently, they can take the measure of the length plus the width and double that amount. They can also calculate two lengths plus two widths to get the perimeter of a rectangle.	The perimeter of the figure above is $2(7 + 5)$ or $2 \times 7 + 2 \times 5$ or, in general, $2(\ell + w)$ or $2\ell + 2w$.
Area of Triangles Students use their knowledge of rectangles to find the area of triangles. If we surround a triangle with a rectangle, we can see that the area of the triangle is half of the area of the rectangle. The triangle may be turned to a convenient side as the base, if needed.	 Sections 1 and 2 are congruent. 3 and 4 are congruent. The area of the triangle is $\frac{1}{2} b \times h$ where b is the base of the triangle (length of the rectangle) and h is the height of the triangle (width of the rectangle).
Perimeter of Triangles Students find the perimeter of a triangle by measuring the lengths of the three sides and adding them together.	The perimeter of the triangle is $7 + 10 + 12.2$, or 19.2 ft. 
Area of Parallelograms Students draw a diagonal creating two congruent triangles. The parallelogram and triangle have the same length of the base and height. Students find the area of the parallelogram by multiplying the base and height, without dividing by two, as they did when finding the area of a triangle.	The area of a parallelogram is the area of two triangles $2 \times (\frac{1}{2} b \times h)$, or just $b \times h$. 
Perimeter of Parallelograms The perimeter of parallelograms is found by measuring the lengths of the four sides and adding them together.	The perimeter of the parallelogram is $2(5 + 6)$ or $2 \times 5 + 2 \times 6 = 22$ cm. 
Area of Circles Students find the number of "radius squares," whose side lengths are equal to the radius, that cover the circle. They find they need a little more than three, or pi.	The area of a circle is pi × a "radius square" or pi × radius × radius = $\pi \times r \times r = \pi r^2$ 
Perimeter of Circles (Circumference) Students count the number of diameter lengths needed to surround the circle. It is a little more than three, or pi.	The circumference of a circle is pi × diameter = πd.

PROBABILITY

Important Concepts	Examples
Probability A number between 0 and 1 that describes the likelihood that an event will occur.	If a bag contains a red marble, a white marble, and a blue marble, then the probability of drawing a red marble is 1 out of 3 or $\frac{1}{3}$. We would write: P(red) $= \frac{1}{3}$.
Once we have a probability—theoretical or experimental—we can use it to make predictions and decisions.	If a number cube is rolled 1000 times, we would predict that a 3 will occur about $\frac{1}{6}$ of the time or about $\frac{1}{6} \times 1000$, or 167 times.
Theoretical Probability A probability obtained by analyzing a situation. If all the **outcomes** (possible results) are equally likely, you can find a theoretical probability of an event by first listing all the possible outcomes, then finding the ratio of the number of outcomes you are interested in to the total number of outcomes.	If a number cube has six sides with the possible outcomes of rolling: 1, 2, 3, 4, 5, or 6, then the probability of rolling a "3" is 1 out of 6. $\text{P(Rolling a 3)} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$ $= \frac{\text{1 (there is 1 number 3 on the cube)}}{\text{6 (there are 6 possible outcomes)}}$
Experimental Probability A probability found as a result of an experiment. This probability is the relative frequency of the **event** (a set of outcomes) that is the ratio of the number of times the event occurred compared to the total number of **trials** (one round of an experiment).	If you tossed a coin 50 times and heads occurred 23 times, the relative frequency of heads would be $\frac{23}{50}$. $\text{P(heads)} = \frac{\text{number of times the event occurred}}{\text{number of trials}}$ $= \frac{\text{number of heads}}{\text{total number of tosses}} = \frac{23}{50}$
Random Events In mathematics, random means that any particular outcome is unpredictable, but the long-term behavior exhibits a pattern.	Flipping a coin is a random event because we never know whether the next flip will be heads or tails, but we do know that in the long run we will have close to 50% heads.

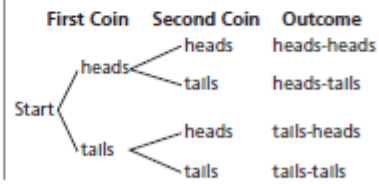
Strategies for Finding Outcomes

When situations involve more than one action, we need to generate the outcomes in a systematic way. Organized lists or tree diagrams are particularly useful.

Organized List

First Coin	Second Coin	Outcome
heads	heads	heads-heads
heads	tails	heads-tails
tails	heads	tails-heads
tails	tails	tails-tails

Tree Diagram



Law of Large Numbers

Experimental data gathered over many trials should produce probabilities that are close to the theoretical probabilities. This idea is sometimes called the Law of Large Numbers.

It is important for students to realize that a small amount of data may produce wide variation. It takes many trials to make good estimates for what will happen in the long run.

The Law of Large Numbers does not say that when flipping a coin, we should expect exactly 50% heads in any given large number of trials. Instead, it says that as the number of trials gets larger, we expect the percentage of heads to be in a smaller range of around 50%.

STATISTICS

Important Concepts and Examples

Representing Data Distributions and Reading Data Representations

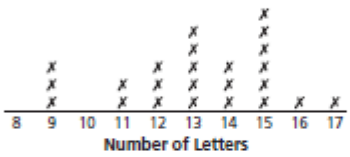
Statisticians use data representations such as line plots, bar graphs, stem-and-leaf plots, and coordinate graphs to describe and analyze their data.

READING STANDARD DATA REPRESENTATIONS

- *Reading the data* involves "lifting" information from a graph to answer explicit questions.
- *Reading between the data* includes the interpretation and integration of information presented in a graph.
- *Reading beyond the data* involves extending, predicting, or inferring from data to answer implicit questions.

LINE PLOT Each case is represented as an "X" positioned over a labeled number line.

Name Lengths of Ms. Jee's Students



STEM-AND-LEAF PLOT
A plot that permits students to group data in intervals (usually by 10s).

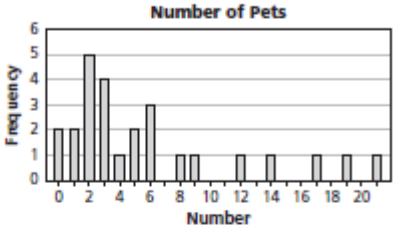
Movies Watched

0	3 3 5 6 6 7
1	1 5 6 8
2	

Key: 1 | 5 means 15 movies

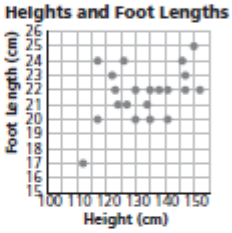
FREQUENCY BAR GRAPH
A bar's height is not the value of an individual case but rather the number (frequency) of cases that all have that value.

Number of Pets



SCATTERPLOT
The relationship between two variables is explored by plotting data values on a Cartesian coordinate system.

Heights and Foot Lengths



Using Measures of Center (Mode, Median, Mean)

MODE The mode is the value that occurs with greatest frequency in a set of data.

14 students said that they had the following number of siblings: 0, 0, 0, 1, 1, 1, 2, 2, 2, 2, 2, 3, 5, 6. The mode is 2.

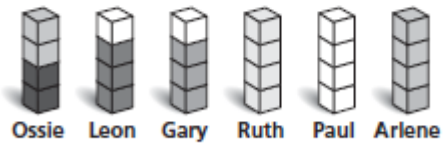
MEDIAN The median value marks the location that separates an ordered set of data in half.

The median for the data set 3, 4, 4, 7, 8, 9 is 5 , the number halfway between 4 and 7. For 4, 5, 5, and 7, the median is 5.

MEAN We emphasize the fair share (or evening out) interpretation of mean (average).

The mean (average) number of people in these households is 4. There are 24 people shared among 6 households.

	BEFORE		AFTER
Ossie	2 people	Ossie	4
Leon	3 people	Leon	4
Gary	3 people	Gary	4
Ruth	4 people	Ruth	4
Paul	6 people	Paul	4
Arlene	6 people	Arlene	4
Total	24 people		24 people



Using Measures of Variability

Measures of variability are used to describe how widely spread or closely clustered the individual data values are.

RANGE The range depends on only two values, the greatest and the smallest.

Distinguishing Different Types of Data

NUMERICAL DATA are values that are counts or measures (pulse, height). We can use mean, median, mode, and range as summary statistics.

CATEGORICAL DATA are data sets that are responses representing categories (favorite color, month of birth, etc.). We can use only the mode as the summary statistic.