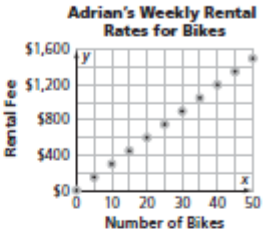
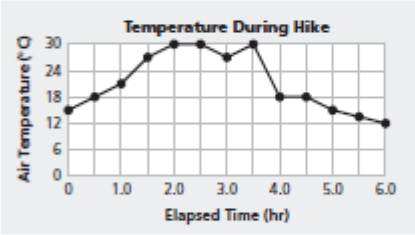
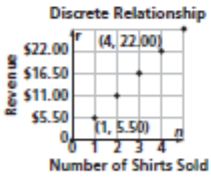
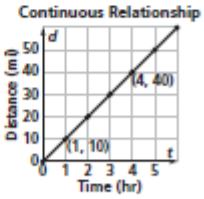
Whiz Kids²

7th GRADE MATH CONCEPTS

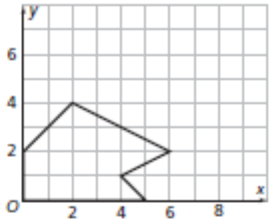
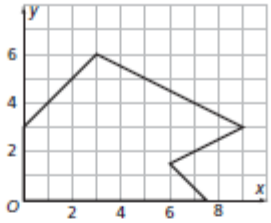
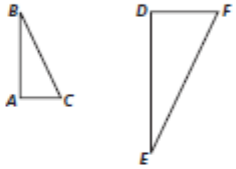
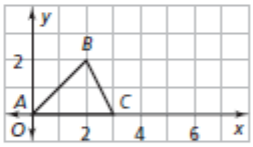
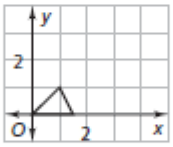
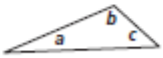
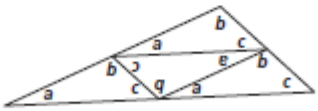
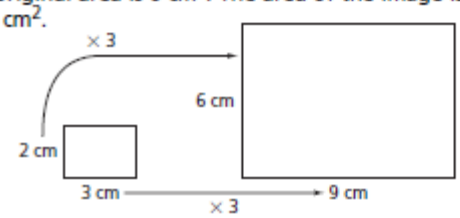
School districts have adopted the Connected Math curriculum and the concepts below are directly in line with that curriculum.

Another helpful resource is www.KhanAcademy.com where you can watch videos explaining each of the following concepts.

Variables and Patterns

Important Concepts	Examples
Variables A variable is a quantity that can change. Letters are often used as symbols to represent variables in rules that describe patterns.	The *number of students, n,* who go on a trip is related to the *price of the trip, p,* for each student.
Patterns A change that occurs in a predictable way. The problems in this unit require students to investigate the patterns of change in values of one variable in relation to changes in value of the other variable.	As the *number of bikes* increases by one, the *rental fee* increases by $30. 
Tables A list of values for two or more variables that shows the relationship between them. The table shows how a change in one variable affects the change in the other variable. The table may show a pattern of change.	As the *number of campsites, x,* changes by one unit, the *total campground fee, y,* changes by 12.5 units. The table can be continued by adding 1 to the previous entry in the *x* row and 12.5 to the previous entry in the *y* row. **Campground Fees** Number of Campsites: 1, 2, 3, 4, 5, 6, 7, 8 Total Campground Fee: $12.50, $25.00, $37.50, $50.00, $62.50, $75.00, $87.50, $100.00
Coordinate Graphs A representation of pairs of related numerical values that show the relationship between two variables. It relates the independent variable (shown on the *x*-axis) and the dependent variable (shown on the *y*-axis). Graphs are another way to view patterns of change.	
Discrete vs. Continuous Data There are two basic types of quantitative variables—those with only a countable set of values (discrete data) and those with any real number values (continuous data). Tables can only represent discrete collections of (*x, y*) values. Graphs can represent both but often suggest continuous variables.	The *number of shirts sold* and *revenue* is a discrete relationship. Connecting two points does not make sense. It would imply that part of a shirt could be sold.   Situations such as the distance/time/rate relation are continuous. If a bicyclist peddles at a rate of 10 miles per hour, it is reasonable to connect the points, because you can go a distance in part of an hour.
Rules and Equations Rules are a summary of a predictable relationship that tells how to find the values of a variable. A rule may be given in words or as an equation. Equations (or formulas) are rules containing variables that represent a mathematical relationship.	Rule (in words): Total profit equals profit per T-shirt times the number of shirts sold Rule (written as an Equation): $y = 10x$ A formula or equation for finding the area of a circle: $A = \pi r^2$

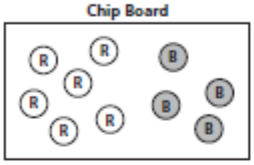
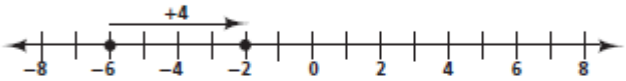
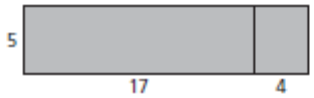
Similarity in Geometry

Important Concepts	Examples
Similarity Two figures are similar if: (1) the measures of their **corresponding** angles are equal and (2) the lengths of their **corresponding** sides are related by the same factor, called the **scale factor**.	The Figures A and B below are similar.  Figure A  Figure B The corresponding angle measures are equal. The side lengths from Figure A to Figure B grow by a factor of 1.5—each side length from A to B is 1.5 times as long. So, the scale factor from Figure A to Figure B is 1.5. (The Figure A stretches or is enlarged to figure B.) We could also say the scale factor from Figure B to Figure A is $\frac{1}{1.5}$ or $\frac{2}{3}$. (The figure B shrinks to figure A.)
Corresponding Corresponding sides or angles have the same relative position in similar figures.	 **Corresponding Sides** *AC* and *DF* *AB* and *DE* *BC* and *EF* **Corresponding angles** *A* and *D* *B* and *E* *C* and *F*
Scale Factor The number used to multiply the lengths of a figure to stretch or shrink it to a similar image. A scale factor larger than 1 will enlarge a figure. A scale factor between 0 and 1 will shrink a figure. The scale factor of two similar figures can be found by a ratio that compares the corresponding sides: $\frac{\text{length of a side on the image}}{\text{length of the corresponding side on the original}}$	If we use a scale factor of $\frac{1}{2}$, all lengths in the image are $\frac{1}{2}$ as long as the corresponding lengths in the original.   The base of the original triangle is 3 units. The base of the image is 1.5 units. The scale factor is $\frac{1.5}{3} = \frac{3}{6}$ or $\frac{1}{2}$.
Area and Scale Factor Lengths of similar figures will stretch (or shrink) by a scale factor. Areas of the figures will not change by the same factor.	If we apply a scale factor of 2 to a figure, the area becomes 4 times as large.  original  four copies If we apply a scale factor of 3 to a figure, the area becomes 9 times as large. The original area is 6 cm^2. The area of the image is 9 times as large or 54 cm^2. 

Ratio, Proportion and Percent

Important Concepts	Examples
Ratio A comparison of two quantities.	*For every 2 cups of mix, you use 3 cups of water.* Ratios are written in several forms: 2 to 3, or 2:3, or $\frac{2 \text{ cups mix}}{3 \text{ cups water}}$.
Ratios in Fraction Form Ratios are often written in fraction form but do not represent fractions. Fractions have part-to-whole comparisons. Some ratios are part-to-part comparisons.	The statement "the ratio of boys to girls in a class is 15 girls to 9 boys" can be written as a fraction, $\frac{15}{9}$, but it does not mean that the fraction of students in the class that are girls is $\frac{15}{9}$. The total number of students in the class is needed. The sum of the numbers of boys and girls is 24. The part-to-whole comparison is $\frac{15}{24}$. So the fraction of the class that is girls is $\frac{15}{24}$.
Ratios as Percents When the ratio can be thought of as part of a whole you can write a percent comparison statement.	*The ratio of concentrate to water in a mix is 3 cups concentrate to 16 cups water. First, find the total cups the recipe makes, 19 cups.* Then, write the fraction of the mix that is concentrate, $\frac{3}{19}$. To find the percent, we divide the concentrate by the total mix, $3 \div 19 = 0.15789\ldots$ or about 15.8% concentrate.
Rate A statement that compares two different variables.	miles to gallons, sandwiches to people, dollars to hours
Unit Rate You have two options when you divide two numbers. The units help you think through the situations so that you can use either set of unit rates to compare the quantities	*Sascha goes 6 miles in 20 minutes on leg 1 of his bike ride. On leg 2 he goes 8 miles in 24 minutes. On which leg is he faster?* $\frac{6 \text{ miles}}{20 \text{ minutes}} = 0.3$ miles per minute and $\frac{8 \text{ miles}}{24 \text{ minutes}} = 0.333$ miles per minute Now the comparison is clear. The times are the same, 1 minute, and the distances can be directly compared. 8 miles in 24 minutes is faster. But, we could divide the other way: $\frac{20 \text{ minutes}}{6 \text{ miles}} = 3.333$ minutes per mile and $\frac{24 \text{ minutes}}{8 \text{ miles}} = 3$ minutes per mile We see that the smaller number tells the correct answer, 8 miles in 24 minutes.
Scaling Ratios (and Rates) Write the ratios as fractions to help the thinking needed for scaling the ratios up (or down). However, we must recognize the difference between dealing with a fraction and dealing with a ratio written as a fraction.	*Which is less expensive, 3 roses for \$5 or 7 roses for \$9?* If we want to scale the costs to be the same, we can use the same thinking as that for finding a common denominator. We would look for a common multiple of 5 and 9. $\frac{3}{5} = \frac{3 \times 9}{5 \times 9} = \frac{27 \text{ roses}}{\$45}$ and $\frac{7}{9} = \frac{7 \times 5}{9 \times 5} = \frac{35 \text{ roses}}{\$45}$. The second option gives more for the same amount of money.
Proportions A proportion is a statement of equality between two ratios. If one part is unknown, we can use scaling or equivalent fractions to find the missing part of a proportion.	*It takes Glenda 70 steps on the elliptical machine to go 0.1 of a mile. When her workout is done, she has gone 3 miles. How many steps has she made on the machine?* $\frac{70 \text{ steps}}{0.1 \text{ miles}} = \frac{x \text{ steps}}{3 \text{ miles}} = \frac{70 \times 30 \text{ steps}}{0.1 \times 30 \text{ miles}} = \frac{2{,}100 \text{ steps}}{3 \text{ miles}}$ Glenda took 2,100 steps to go 3 miles.

Positive and Negative Numbers

Important Concepts	Examples
Negative Numbers Negative Numbers are the opposites of positive numbers. **INTEGERS** are the set of the whole numbers and their opposites. **RATIONAL NUMBERS** are the positive and negative integers and fractions.	Negative numbers: $-\frac{2}{3}$, -24, -1 Integers: -14, -29, 0 Rational numbers: -2, $-1\frac{2}{3}$, 0, $\frac{3}{4}$, 14
Addition and Subtraction Students model and symbolize problems to develop meaning and skill in addition and subtraction before developing algorithms. The **colored chip model** requires an understanding of opposites. For example, 4 black chips represent $+4$ and 4 red chips represent -4. $4 + (-4) = 0$ because $+4$ and -4 are opposites. The **number line model** helps make the connection to rational numbers as quantities. Sometimes it is helpful to restate an addition problem as a subtraction or a subtraction problem as an addition.	Chip Board  *Johnson owed his sister $6.00. He earned $4.00 delivering papers. What is his net worth?* One color chip (black) represents positive numbers and another chip (red) represents negative numbers. Collections of black and red chips on a board represent the combination of expense and income. The result, or net worth, is that he is "in the red" 2, or $^{-}2$ dollars. This problem may be represented with the number sentence $^{-}6 + {}^{+}4 = {}^{-}2$.  To calculate $^{+}12 + {}^{-}8$, the result is the same as if you subtract $^{+}8$ in the problem, $^{+}12 - {}^{+}8$. To calculate $^{+}5 - {}^{-}7$, the result is the same as if you add $^{+}7$ in the problem $^{+}5 + {}^{+}7$.
Multiplication Multiplication can be modeled using a number line model and "counting" occurrences of fixed-size movement along the number line.	$8 \times (-6)$ This can be represented as 8 jumps of $^{-}6$ on the number line. $^{-}6 + {}^{-}6 + {}^{-}6 + {}^{-}6 + {}^{-}6 + {}^{-}6 + {}^{-}6 + {}^{-}6 =$ $^{-}48$ or $8 \times {}^{-}6 = {}^{-}48$
Division A multiplication fact can be used to write two related division facts.	We know that $5 \times {}^{-}2 = {}^{-}10$. Write the related division sentences: ${}^{-}10 \div {}^{-}2 = 5$ and ${}^{-}10 \div 5 = {}^{-}2$. From this relationship students can determine the answer to a division problem.
Order of Operations Mathematicians have established rules for the order in which operations ($+, -, \times, \div$) should be carried out.	**1.** Do any computations in parentheses. $3 + 4 \times \underline{(6 \div 2)} \times 5 - 7^2 + 6 \div 3 =$ $3 + 4 \times 3 \times 5 - \underline{7^2} + 6 \div 3 =$ **2.** Compute exponents. $3 + \underline{4 \times 3} \times 5 - 49 + 6 \div 3 =$ **3.** Do all $\div$ or $\times$ operations in order from left to right. $3 + \underline{12 \times 5} - 49 + \underline{6 \div 3} =$ $\underline{3 + 60} - 49 + 2 =$ **4.** Do all $+$ or $-$ operations in order, from left to right. $\underline{63 - 49} + 2 =$ $\underline{14 + 2} = 16$
Commutative Property The order of addends does not matter. The order of factors does not matter. Subtraction and division do not have this property.	$5 + 4 = 4 + 5$ $\quad -2 + 3 = 3 + -2$ $5 \times 4 = 4 \times 5$ $\quad -2 \times 3 = 3 \times -2$ $5 - 4 \neq 4 - 5$ $\quad -2 - 3 \neq 3 - -2$ $5 \div 4 \neq 4 \div 5$ $\quad -2 \div 3 \neq 3 \div -2$
Distributive Property The distributive property shows that multiplication *distributes* over addition. This property is introduced and modeled through finding areas of rectangles.	$5 \times (17 + 4) = (5 \times 17) + (5 \times 4)$ 

Linear Relationship

Important Concepts and Examples

Linear Relationships

A relationship is linear if there is a constant rate of change between the two variables. That is, for each unit change in x there is a constant change in y.

TABLES In the table the **constant rate of change** can be observed as a pattern of consistent change in the variables.

Gilberto's Walking Rate

Time (seconds)	Distance (meters)
0	0
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18

+2 +4 +3 +6

As time increases by 1 second, the distance increases by 2 meters. The constant rate of change is 2 meters per second.

GRAPHS If we graph the data, the constant rate of change between the two variables shows up as a straight line.

This constant rate of change is called the **slope of the line**. It is the ratio of change between the two variables.

$$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}},$$

for any two points on the line.

Here, the slope is $\frac{4}{2}$ or $\frac{6}{3}$ or $\frac{2}{1}$.

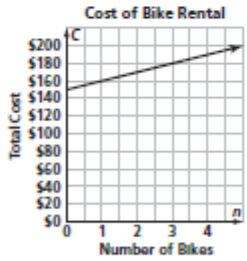
EQUATIONS In this symbolic representation the constant rate of change shows up as the **coefficient** of t.

$d = \mathbf{2}t$ (Gilberto)

Here, the coefficient is 2.

***y*-INTERCEPT** The point where the graph of a line crosses the y-axis (vertical axis).

Suppose the cost to rent bikes is represented by $C = \$150 + \$10n$, where C is the cost in dollars and n is the number of bikes.



The y-intercept is \$150.

The y-intercept is the constant term in the equation, $C = 150 + 10n$.

The slope (or the constant rate of change) of the line is 10.

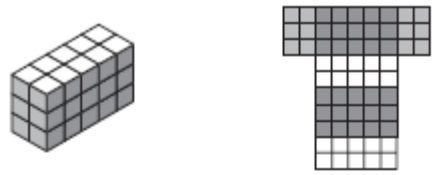
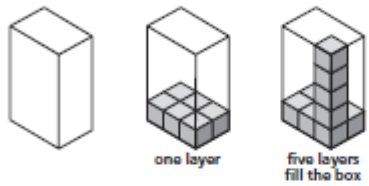
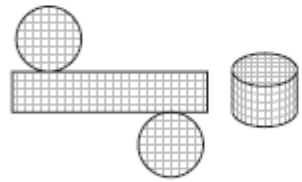
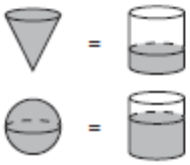
Solving Equations

Write a series of equivalent equations until it is easy to read the value of the variable. Equality or equivalence is maintained when you add, subtract, multiply, and divide the same quantity to each side of the equation. These procedures are called the *properties of equality*.

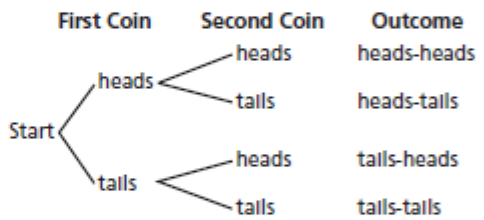
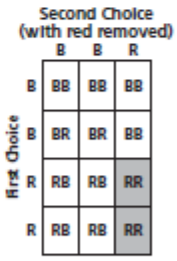
EQUATION	REASONS
$750 = 150 + 10n$	Original equation
$750 - 150 = 150 - 150 + 10n$	Undo "adding 150" by subtracting 150 from each side.
$600 = 10n$	
$\frac{600}{10} = \frac{600n}{10}$	Undo "multiplying by 10" by dividing by 10.
$60 = n$	The value of n must be 60.

Note that if n is replaced by 60 in each step, we have a true equation. The original equation would give $750 = 750$.

Three-Dimensional Measurement

Important Concepts	Examples
Surface Area of Rectangular Prisms To find the surface area of a box (prism), determine the total area needed to wrap the container. Nets can represent boxes. The area of the net is the surface area of the box. Surface area is the sum of the areas of the faces. Surface Area = (area of the front × 2) + (area of the side × 2) + (area of the top × 2) or Surface Area = (area of the front + area of the side + area of the top) × 2 = $(w \times h + w \times \ell + \ell \times h) \times 2$.	 There are two of each of these faces of the prism: 2 cm by 3 cm (area is 6 sq. cm.); 2 cm by 5 cm (area is 10 sq. cm.); 3 cm by 5 cm (area is 15 sq. cm.). Surface area = 62 sq. cm.
Volume of Rectangular Prisms To find the volume of a rectangular box, count the number of layers of unit cubes it takes to fill the container. The number of unit cubes in one layer is equal to the area of the base. The volume (the total number of unit cubes) of a rectangular prism is the area of its base (the number of unit cubes in the first layer) multiplied by its height (the total number of layers). Volume = Area of the base × height = $Bh = \ell wh$.	 one layer five layers fill the box 2 × 3 = 6 cubes on the area of the base 5 stacks of cubes in the height Volume = 6 × 5 = 30 cubic units
Volume of Prisms The volume of any prism is the area of its base multiplied by its height. Volume = Area of the base × height = Bh.	 Rectangular Prism Triangular Prism Pentagonal Prism
Surface Area of Cylinders The surface area of the cylinder is the area of the rectangle that forms the lateral surface ($2\pi rh$) plus the areas of the two circular ends ($2\pi r^2$) where r = radius and h = height. Surface Area = $2\pi r^2 + 2\pi rh$.	 Use 3.14 for π. $r = 4$ $h = 5$ $2(\pi 4^2) + 2\pi 4(5) \approx$ 100.48 + 125.6 = 226.08 square units
Volume of Cylinders The volume of a cylinder is the number of unit cubes in one layer (the area of the circular base) multiplied by the number of layers (the height) needed to fill the cylinder. The area of the base (πr^2) is multiplied by the height to find the volume. Volume = $Bh = \pi r^2 h$	$r = 1.5$ $h = 7$ Area of the base ≈ 3.14 × 2.25 = 7.065 Volume ≈ 7.065 × 7 = 49.455 cubic units
Volume of Cones and Spheres When all three have the same radius and the same height, 1 cone fills $\frac{1}{3}$ of a cylinder, and 1 sphere fills $\frac{2}{3}$ of a cylinder. Cone Volume = $\frac{1}{3}$ volume of the cylinder = $\frac{1}{3}\pi r^2 h$ Sphere Volume = $\frac{2}{3}$ volume of the cylinder = $\frac{2}{3}\pi r^2 h$	Volume of the cylinder = 628 cm^3.  Volume of the cone is approximately 209 cm^3. Volume of the sphere is approximately 419 cm^3.

Probability

Important Concepts	Examples
Probability A number between 0 and 1 that describes the likelihood that an event will occur.	If a bag contains a red marble, a white marble, and a blue marble, then the probability of drawing a red marble is 1 out of 3 or $\frac{1}{3}$. We would write: P(red) = $\frac{1}{3}$.
Theoretical Probability If all the **outcomes** are equally likely, you can find the theoretical probability of the event by first listing all the possible outcomes, then find the ratio of the number of outcomes of interest to the total number of outcomes.	If a number cube has six sides with the possible outcomes of rolling: 1, 2, 3, 4, 5, or 6, the probability of rolling a "3" is 1 out of 6. $P(\text{Rolling a 3}) = \frac{\text{number of equally likely favorable outcomes}}{\text{total number of equally likely outcomes}} = \frac{1\ (\text{there is 1 number 3 on the cube})}{6\ (\text{there are 6 possible outcomes})}$
Experimental Probability This probability is the relative frequency of the **event**. It is the ratio of the number of times the event occurred compared to the total number of **trials**.	If you tossed a coin 50 times and heads occurred 23 times, the relative frequency of heads would be $\frac{23}{50}$. $P(\text{heads}) = \frac{\text{number of times the event occured}}{\text{number of trials}} = \frac{\text{number of heads}}{\text{total number of tosses}} = \frac{23}{50}$
Random Events Outcomes that are uncertain when viewed individually, but which exhibit a predictable pattern over many trials are random.	Rolling a fair number cube is random because although you have no way of knowing what the next roll will be, you do know that, over the long run, you will roll each number on the cube about the same number of times.
Tree Diagram This is a diagram used to determine the number of possible outcomes. The number of final branches is equal to the number of possible outcomes.	
Area Model This is a diagram in which fractions of the area correspond to probabilities in a situation. Area models are particularly helpful when there are 2 events to track and the outcomes of each event are not equally likely.	The area model here shows the probability of getting two red blocks if there are 2 red blocks and 2 blue blocks and one is drawn at a time without replacing it. The probability is $\frac{2}{12}$ or $\frac{1}{6}$. 
Expected Value or Long-term Average The average result over many trials is the expected value. A player's average score per shot in a 1–and–1 free throw situation is an expected value. A player's average winning per lottery ticket is also an expected value.	A game is played with two number cubes. You score 2 points when a sum of 6 is rolled, 1 point for a sum of 3, and 0 points for anything else. If you roll the cubes 36 times, you could expect to get a sum of 6 about five times and a sum of 3 twice. You could expect to score $(5 \times 2) + (2 \times 1) = 12$ points for 36 rolls, an average of $\frac{12}{36} = \frac{1}{3}$ point per roll. This is the expected value of one roll.
Law of Large Numbers Experimental data gathered over many trials should produce probabilities that are close to the theoretical probabilities.	For 1 million flips, exactly 50% heads is improbable, but it would be extremely unlikely for the percent heads to be very different from 50%.

Statistics

Important Concepts and Examples

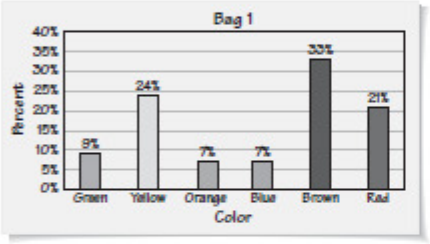
Representing Data Distributions

Statisticians use representations or summary statistics during the analysis part of the process of statistical investigation to describe the data distribution.

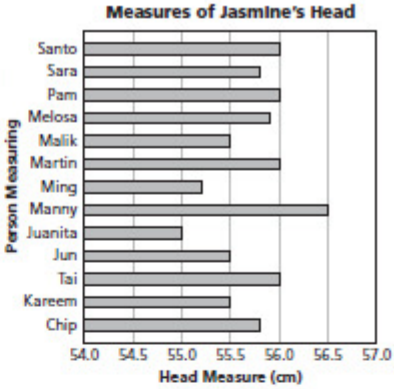
READING STANDARD DATA REPRESENTATIONS

- Reading the data involves "lifting" information from a graph to answer explicit questions.
- Reading between the data includes the interpretation and integration of information presented in a graph.
- Reading beyond the data involves extending, predicting, or inferring from data to answer implicit questions.

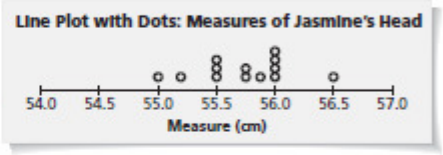
FREQUENCY BAR GRAPH A bar's height is the number (frequency) of cases that all have that value.



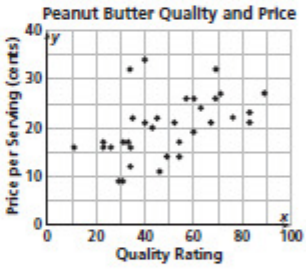
VALUE BAR GRAPH
Each case is represented by a separate bar whose relative length corresponds to the magnitude or value of that case.



DOT PLOT (OR LINE PLOT)
Each case is represented as a dot (or an "x") positioned over a labeled number line.



SCATTERPLOT
The relationship between two different attributes is explored by plotting values of two numeric attributes on a Cartesian coordinate system.



Measures of Central Tendency or Location (Mode, Median, Mean)

MODE The data value or category that occurs with greatest frequency. It is not usually used for summarizing numerical data.

Number of siblings: 0, 0, 0, 1, 1, 1, 2, 2, 2, 2, 2, 3, 5, 6.

The mode is 2.

MEDIAN The numerical value that marks the middle of an ordered distribution. It is not influenced by extreme data values. Graphically, the median marks the location that divides a distribution into two equal parts.

The median for the data set 3, 4, 4, 7, 8, 9 is $5\frac{1}{2}$, the number halfway between 4 and 7.

For 4, 5, 5, and 7, the median is 5.

MEAN The numerical value that marks the balance point of a distribution; it is influenced by all values of the distribution including extremes and outliers. It is a good measure to use when working with distributions that are roughly symmetric.

Number of people in household: 2, 3, 3, 4, 6, 6.

The mean (average) number of people in these households is 4. There are 24 people "shared" among 6 households.

Using Measures of Variability

Measures of variability are used to describe how widely spread or closely clustered the individual data values are.

Range depends on two values, the greatest and the smallest. Range is the difference between the greatest value and the least value in the data.